Purpose: In this problem set, you will begin to explore logarithmic functions through algebraic manipulations.

1. Recall the function $f(x) = b^x$, where $b > 0$.

   (a) Give the domain and range of $f(x)$.

   (b) Is f one-to-one? (I recommend sketching a quick graph.)

   (c) What is the inverse of f?

   (d) What is the inverse of $y = 2^x$?

   (e) What is the inverse of $y = \left(\frac{1}{3}\right)^x$?

   (f) What is the inverse of $y = 10^x$?

Definition: The **logarithm function** (of base b), written $\log_b(x)$, is the inverse of the the exponential function (of base b), b^x.

Since these two functions are inverses, we get the **inverse properties of logs**:

We also get a wildly useful equivalence statement:

2. Before we get too far, let's look at some applications.

   (a) Chemists define the acidity or alkalinity of a substance by the formula $pH = -\log(\mathbf{H}^+)$ where $\mathbf{H}^+$ is the concentration of hydrogen ions in moles per liter. Solutions with a pH value of less than 7 are acidic; solutions with a pH value of greater than 7 are basic; solutions with a pH of 7 (such as pure water) are neutral.

      i. Which of the following tables represents pH given hydrogen ion concentration?

pH	0	1	2	3	4
$\mathbf{H}^+$ (moles/liter)	1	1/10	1/100	1/1,000	1/10,000

pH	0	1	2	3	4
$\mathbf{H}^+$ (moles/liter)	0	1/10	1/20	1/30	1/40

      ii. Is a pH of 3.5 closer to a concentration of $\dfrac{1}{1,000}$ or $\dfrac{1}{10,000}$?

(b) "Loudness" is measured in decibels. The formula for the loudness of a sound is given by $dB = 10 \log(I \div I_0)$ where I_0 is the intensity of "threshold sound", or sound that can barely be perceived. Other sounds are defined in terms of how many times more intense they are than threshold sound. For instance, a cat's purr is about 316 times as intense as threshold sound.

Find the decibel rating of a cat's purr.

(c) If \$1,000 is invested in an account earning 2% APR compounded quarterly, how long will it take the account to grow in value to \$1,300?

3. Recall that exponential functions grow faster than EVERY polynomial! What do you predict about the growth rate of their inverses, the logarithms? Use the space below to state your prediction and sketch graphs that help you test your prediction.

FACT:

4. Using our review from the start of class and our knowledge of inverse functions, answer the questions below:

   (a) What is the range of a logarithmic function?

   (b) What is the domain of a logarithmic function?

   (c) What does the graph of a logarithmic function look like?

5. As somewhat of an aside, we have a new domain restriction for functions! That makes three.

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6. Find the domain of $\log_2(3x+1)$.

7. Find the domain of $\log_3(x^2-3x+4)$.

8. Find the domain of $\log_{10}\left(x^3-x\right)$.